HI Michelle,

The Sept/Oct 2025 QEX pp. 19-22 has a number of mistakes that might have crept into the article during the editorial process. As a lifelong DSP learner, and as a voracious book reader, I realize how difficult it is to write about DSP! I share as a DSP tutor who helps colleagues unwind things they read, or wrote, that were not quite true. Fire back on my mistakes too. That is how we learn!

(A) **QEX Page 19:** "Let's filter out the lower image, and transmit f_c, which is our modulated signal at the carrier frequency." This statement if false. The problem is that Euler's Identity for cosine expresses the two images you referenced [1]:

$$\alpha \, cos\left(2\pi f_c t\right) = \alpha \, \frac{e^{i2\pi f_c t} + e^{-i2\pi f_c t}}{2}$$

You cannot transmit only the upper image because of Euler's equation [2]. Your "upper image" is only:

$$\frac{\alpha}{2} e^{i2\pi f_c t} = \frac{\alpha}{2}\left(cos\left(2\pi f_c t\right) + i \, sin\left(2\pi f_c t\right)\right).$$

The problem is that this signal is <u>complex-valued</u> now. For example:

$$Re\left\{\frac{\alpha}{2} e^{i2\pi f_c t}\right\} = \frac{\alpha}{2} cos\left(2\pi f_c t\right)$$

$$Im\left\{\frac{\alpha}{2} e^{i2\pi f_c t}\right\} = \frac{\alpha}{2} sin\left(2\pi f_c t\right)$$

Since neither the real nor the imaginary part is zero, when you deleted your "lower image" you have produced a complex-valued signal. You cannot transmit this signal using a single antenna since it is complex-valued now!

(B) **QEX Page 20:** "Adding the I and Q signals together and transmitting them sends $I\,cos\left(2\pi f_c t\right) + Q\,sin\left(2\pi f_c t\right)$." That is unconventional. Although it may surprise many, most textbooks actually use this form instead [3,4]:

$$I\,cos\left(2\pi f_c t\right) - Q\,sin\left(2\pi f_c t\right).$$

Most books use a negative on the sinusoid component because it is the only form that leads to consistency with other DSP equations — *namely the Fourier transform!* As students learn DSP, eventually they encounter phase modulation where the I & Q values are set according to an angle θ:

$$I = cos\left(\theta\right) \quad \text{and} \quad Q = sin\left(\theta\right).$$

Most textbooks use the negative sinusoid since:

$$cos\left(\theta\right) cos\left(2\pi f_c t\right) - sin\left(\theta\right) sin\left(2\pi f_c t\right) = cos\left(2\pi f_c t + \theta\right).$$

One problem — *there are many* — of using the positive sinusoid is that:

$$cos\,(\theta)\,cos\left(2\pi f_c t\right) + sin\,(\theta)\,sin\left(2\pi f_c t\right) = cos\left(2\pi f_c t - \theta\right).$$ (Bad Convention!)

Students run into trouble with this form when they encounter frequency modulation. A positive change in phase with respect to time should shift the transmitted frequency up, and this is only possible if we use the negative sinusoid of the carrier.

The other problem with your form is inconsistency with Euler and Fourier. For instance, a phase modulated signal in most textbooks is formulated as:

$$Re\left\{\left(I + iQ\right)\left(cos\left(2\pi f_c t\right) + i\,sin\left(2\pi f_c t\right)\right)\right\} = I\,cos\left(2\pi f_c t\right) - Q\,sin\left(2\pi f_c t\right)$$

Notice the negative sign on the right hand side! Now, you will find a few popular textbooks that get this wrong, but we know from algebra that:

$$i^2 = -1.$$

This makes the carrier sinusoid negative in fact! The negative is essential for using all of the famous math results to make the work easier.

(C) **QEX Page 20:** "What happens when we integrate a cosine signal from 0 to T? That value happens to be zero!" This is not true because you stated "the value that we are transmitting is held for a period of time called T" on QEX page 19. Saying this leads to problems for students understanding the DTFT and FFT later. In fact, if we clarify the term T as T_{sym} — *avoiding any ambiguity of reference to a period of the cosine wave* — we have:

$$\int_0^{T_{sym}} cos\left(2\pi\left(2f_c\right)t\right)dt = \frac{sin\left(2\pi\left(2f_c\right)T_{sym}\right)}{4\pi f_c}$$

That is clearly not zero for just any old value of T_{sym}. To see this graphically, check out Figure 3-2 in my textbook that hopefully makes it easier [5]. I suggest qualifying the integration period if you present this material in the future.

(D) **QEX Page 22:** "However, using this complex modulation scheme gives us yet another advantage. Because of the math we just did, we eliminate an entire image when compared to a single carrier real signal." This is not true. Your form equals:

$$I\,cos\left(2\pi f_c t\right) + Q\,sin\left(2\pi f_c t\right)$$

Suppose you sent only a "single carrier real signal" and expand using Euler's Identity for cosine:

$$I\,cos\left(2\pi f_c t\right) = I\,\frac{e^{i2\pi f_c t} + e^{-i2\pi f_c t}}{2}$$

Now, suppose you instead send your entire signal and further expand using Euler's Identity for sine:

$$I\,cos\left(2\pi f_c t\right) + Q\,sin\left(2\pi f_c t\right) = I\,\frac{e^{i2\pi f_c t} + e^{-i2\pi f_c t}}{2} + Q\,\frac{e^{i2\pi f_c t} - e^{-i2\pi f_c t}}{2i}$$

Gather terms to see there is no cancellation of images assured by your operations:

$$I\,cos\left(2\pi f_c t\right) + Q\,sin\left(2\pi f_c t\right) = \left(\frac{I}{2} + \frac{Q}{2i}\right)e^{i2\pi f_c t} + \left(\frac{I}{2} - \frac{Q}{2i}\right)e^{-i2\pi f_c t}$$

From this form, arbitrary values for I & Q do not change the number of images versus your other case.

I am sure I made a few mistakes too, but I wanted to share my insights since it seemed a few things crept into an odd place in this article.

-Pete

References

[1]. T. Needham, Visual Complex Analysis, Oxford University Press, New York, 1997, p. 14.
[2]. T. Needham, Visual Complex Analysis, Oxford University Press, New York, 1997, p. 11-12.
[3]. J.G. Proakis, Digital Communications, McGraw-Hill, Boston, 1995, Equation 4-1-12, p. 155.
[4]. R.E. Blahut, Modem Theory, Cambridge University Press, Cambridge, 2010, p. 131.
[5]. P. Wyckoff, Visualizing Signal Processing with Complex Values, KDP, 2023, p. 39.