Efficiently Using Transmitted Symbol Energy via Delay-Doppler Channels — Part II

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Introduction

Part II builds examples with Delay-Doppler channels from the Part I foundation of multi-path with delay but without Doppler. Part I defined an efficiency metric as the ratio of energy-per-symbol that actually reaches the demodulator versus the energy-per-symbol transmitted [1]. The efficiency metric showed that certain DSP waveform designs transfer energy from the transmitter to the demodulator more efficiently than others when there is multi-path propagation and a known channel response [2]. This effect exploits superposition of fields from two or more propagation paths. Figure 1 illustrates the effect at a receiver's low intermediate frequency (IF) to simplify visualization.

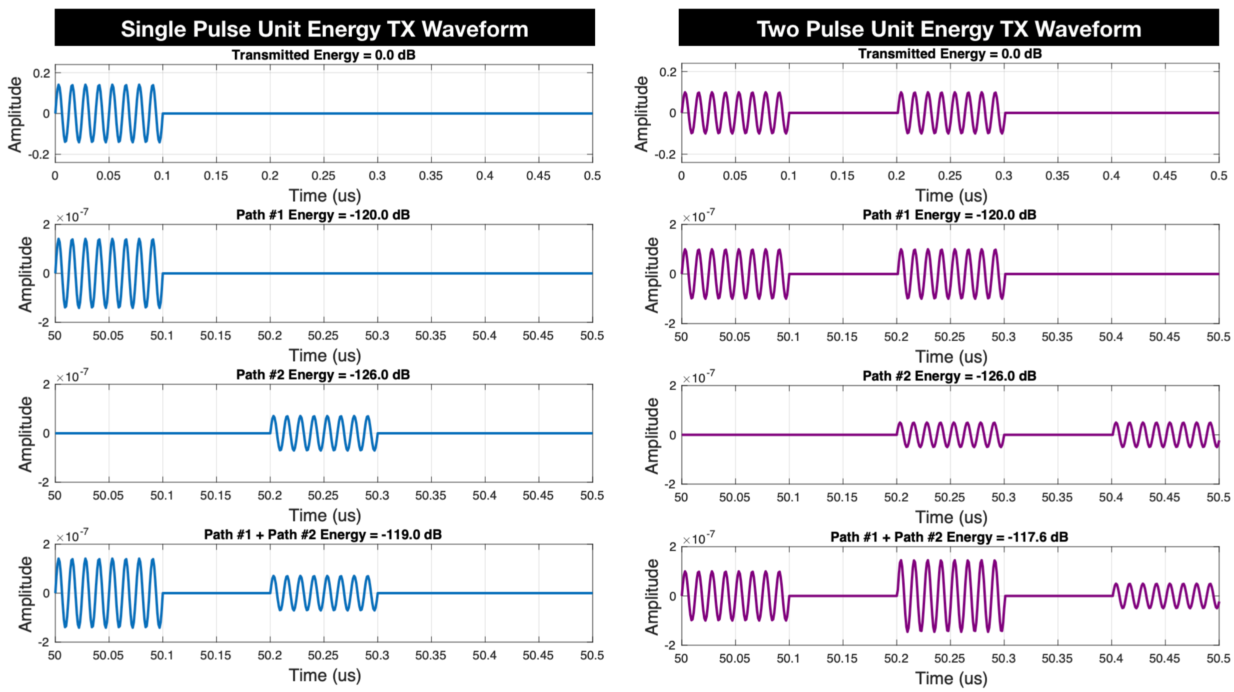

FIGURE 1: (LEFT) A single pulse unit energy transmit waveform propagates to the receiver via two paths and delivers energy -119.0 dB to the receiver. (RIGHT) A two pulse unit energy transmit waveform propagates via the same two paths and delivers energy -117.6 dB to the receiver — *boosting energy transfer from transmitter to receiver by +1.4 dB for this channel*.

The Figure 1 top row shows two alternatives for the transmitted waveform design — *single pulse vs. two pulse waveform*. Crucially, both waveforms have equal transmitted energy-per-symbol. The second row in Figure 1 shows each waveform after propagating via the first path. The third row of Figure 1 shows each waveform after propagating via the second path featuring somewhat greater path delay and further decreased amplitude.

The energy received from each individual path is not affected by using one pulse or two pulses for the transmitted waveform in Figure 1. Rather, superposition of these two paths at the receiver antenna produces the important effect — *see bottom row of Figure 1*. Fields from the two paths combine constructively for the two pulse signal between times 50.2 and 50.3 us. This boosts the overall received energy for the two pulse waveform. In Figure 1, the two pulse waveform transferred energy from the transmitter to the receiver +1.4 dB more efficiently than the single pulse waveform for this channel. Of course, this assumes we have an accurate estimate of the channel and the boost in efficiency would degrade with errors in that estimate.

Some amateur radio channels exhibit not only multi-path with delay, but multi-path with Delay-Doppler. Figure 2 repeats the same experiment with different Doppler on each path. This shows the two pulse waveform design no longer delivers any boost in energy transferred. Both signal designs deliver -119 dB of energy. The reason is that superposition of the two paths no longer fully and constructively interferes for the two pulse waveform between 50.2 and 50.3 us.

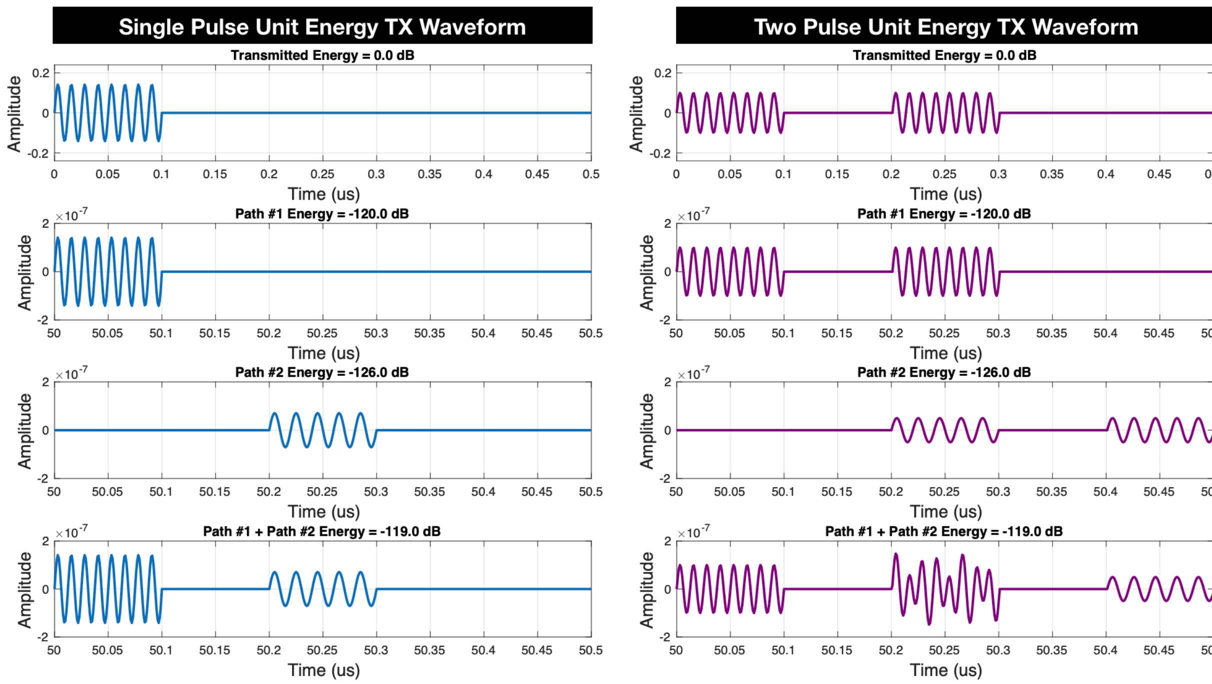

FIGURE 2: Different Doppler on the second path destroys the energy efficiency boost for the two-pulse waveform. Doppler is exaggerated in this diagram to help visualization. Realistic Doppler is addressed later in this paper along with more sophisticated waveform designs.

Improved waveforms to accommodate delay-Doppler channels are the focus of this Part II paper. A simple example resolves the limitation revealed in Figure 2. Then, this foundation is extended to more representative amateur radio channels where Doppler shifts between the different paths might vary by only several Hz over less than a milli-second. Conventional Orthogonal Time-Frequency Space (OTFS) provides inspiration [3]. A new extension of OTFS provides one design methodology to boost the efficiency of energy transfer from transmitter to receiver for such amateur delay-Doppler channels. Finally, performance is characterized with channel estimation errors of varying severity.

Simple Example to Resolve the Doppler Limitation

Different Doppler on the second path will produce fully constructive interference if the two pulses are sent at different center frequencies — *see Figure 3*. This enhanced design restores the two-pulse waveform energy at the receiver to -117.6 dB. The enhanced two pulse waveform is then 1.4 dB more efficient than the single pulse waveform. Incorporating the channel delay and Doppler into the waveform design may increase the energy transferred from the transmitter to the receiver via the delay-Doppler multi-path channel as demonstrated in Figure 3. As in Part I, a waveform with more than two pulses could also be designed to boost efficiency further [4]. Figure 3 merely shows the basic concept for the simplest channel.

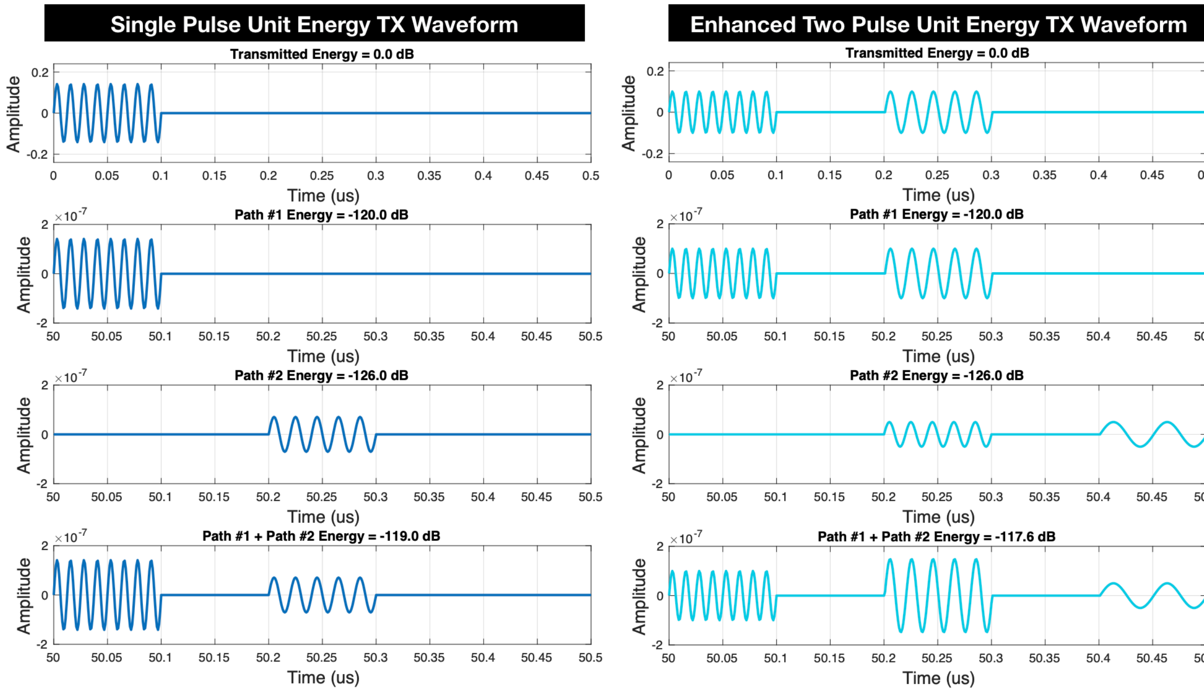

FIGURE 3: The enhanced two-pulse waveform design takes advantage of this known delay-Doppler channel. Transmitting two pulses at different times and frequencies restores the +1.4 dB boost in energy transfer efficiency from the transmitter to the receiver. Doppler is exaggerated in this diagram to help visualization. Realistic Doppler is addressed later in this paper along with more sophisticated waveform designs.

Time-Frequency Perspective on the Two Pulse Waveform & Delay-Doppler Channel

Figure 4 shows a two pulse signal in the time-domain and in the time-frequency domain. The top row shows the transmitted signal from both viewpoints. There are two pulses at the same frequency and separated by 0.2 micro-seconds in time.

The second row in Figure 4 shows the signal that arrives from the first path. It is the transmitted signal shifted in time and reduced in amplitude by 120 dB.

The second path signal appears in the third row of Figure 4. This path is delayed further and reduced in amplitude further. As well, this second path has a Doppler shift with respect to the first path. The time-frequency plot shows this clearly as it resolves the frequency versus time.

The bottom row of Figure 4 shows the superposition of the first and second path in the time domain (lower left) and in the time-frequency domain (lower right). Superposition between 50.2 and 50.3 micro-seconds does not fully constructively interfere because the two paths arrive on two distinct frequencies in Figure 4.

Figure 5 shows the enhanced two pulse signal in time and in time-frequency. The top row of Figure 5 emphasizes the enhanced two pulse signal sends pulses on two different frequencies. The middle two rows show these pulses after traversing the first and second paths. Finally, the bottom row shows this enhanced signal delivers time and frequency alignment of pulses received over the paths from 50.2 to 50.3 micro-seconds.

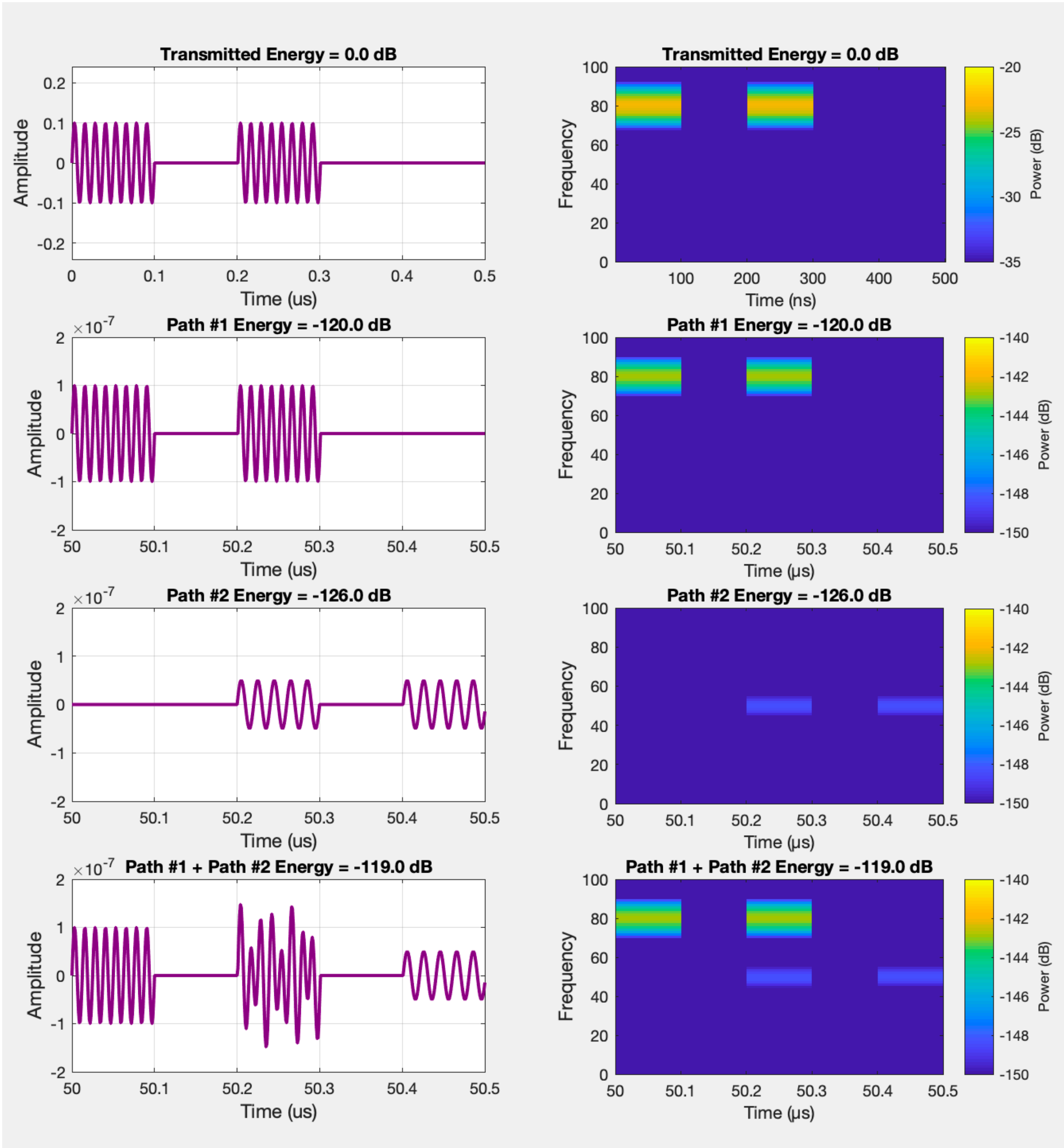

FIGURE 4: Time-frequency perspective shows this transmit waveform design misaligns the pulses in frequency between 50.2 and 50.3 microseconds. There is no boost in energy transfer efficiency since the super-position is not aligned in time and frequency.

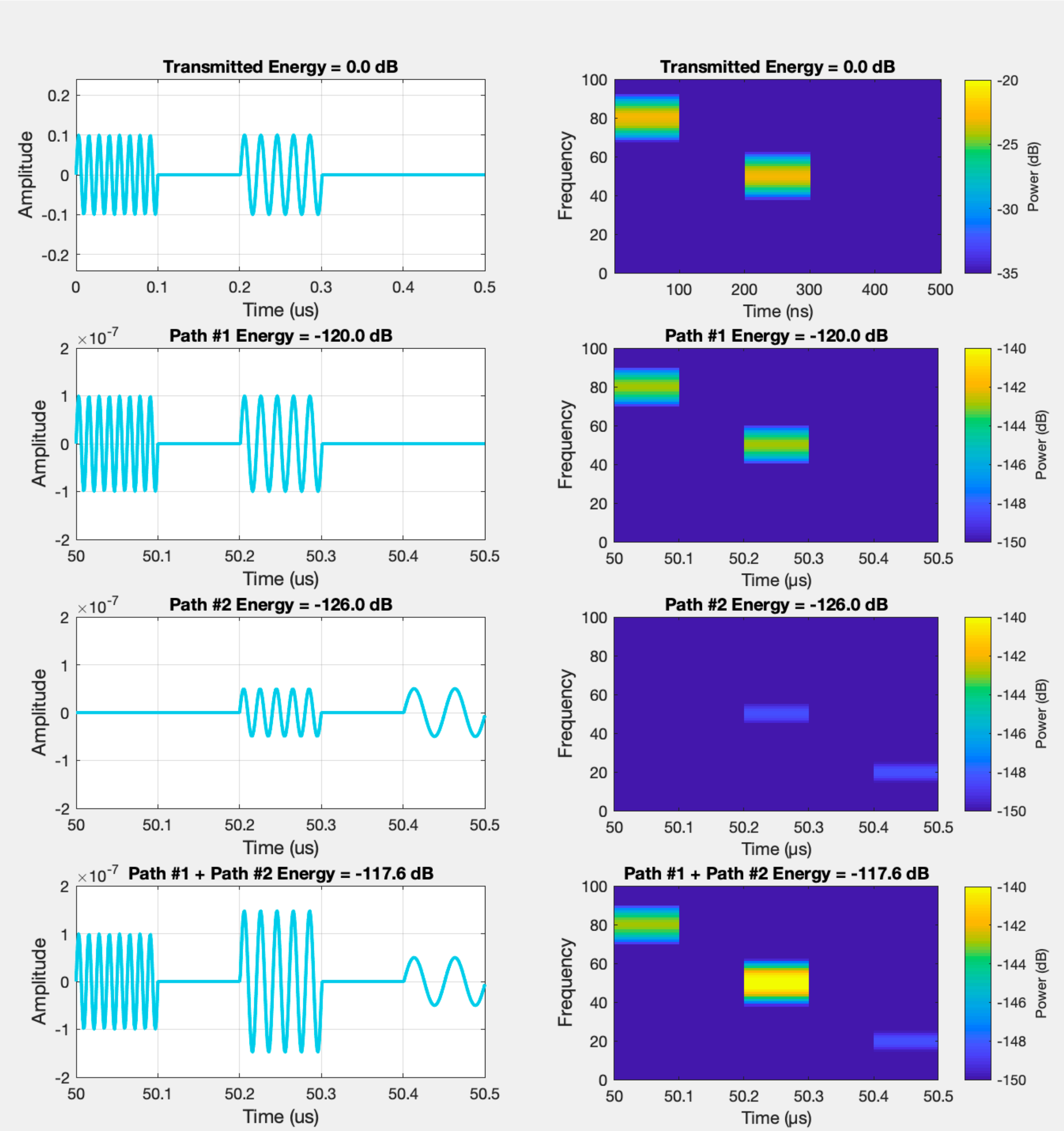

FIGURE 5: Time-frequency perspective shows this transmit waveform design causes a boost in energy delivered to the receiver. The two paths align perfectly between 50.2 and 50.3 micro-seconds, which boosts the received energy.

Extending OTFS Waveforms for More Practical Delay-Doppler Values

This section extends a modulation called orthogonal time frequency space (OTFS) to boost energy efficiency through the multi-path channel. Practically speaking, this is needed because earlier figures exaggerated the Doppler shift between paths and assumed a short symbol period simply to clarify the visualization. That also made the waveform design easier.

More practical constraints require a different approach. Instead of working in time-frequency space, the delay-Doppler space provides a better way. It is only slightly more complicated to boost channel energy transfer efficiency in this manner. The function “*makeOTFS*” creates two series of uniformly spaced pulses that are root-raised cosine filtered and then shifted by a specified Doppler frequency:

```
function tx = makeOTFS(tau, fd, a, Fs, Tsym)
%-----------------------------------------------------------------------
% tx = makeOTFS(tau, fd, Fs, Tsym)
%
% Creates an enhanced OTFS signal designed specifically for the 2-path
% channel.
%
% Variable  Size     I/O  Description
% --------  -------  ---  ---------------------------------------
% tau       1x2      In   Delay for each of two paths in (s)
% fd        1x2      In   Doppler for each of two paths in (Hz)
% a         1x2      In   Complex coef. for each of two paths
% Fs        1        In   Sampling rate in (samples/s)
% Tsym      1        In   Symbol Duration in (s)
% tx        Varies   Out  Transmit signal samples as time-series
%
%                                        -Pete Wyckoff, Version 06172025.
%-----------------------------------------------------------------------
  Tp = max(tau) - min(tau);        %min. time between all path pairs (s)
  T = 2*Tp;                        %prototype pulse period (s)

  for m=1:2                             %two propagation paths
    t = tau(m):T:Tsym;                  %pulse times for mth train (s)
    idx = round(t * Fs) + 1;            %pulse times for mth train (sample)
    x(m,idx) = exp(1i*2*pi*fd(m)*t);    %pulses for mth path
    x(m,idx) = x(m, idx) * exp(-1i*angle(a(m)));  %phase adj.
  end

  [num, den] = rcosine(1/Tp, Fs, 'sqrt', 0.4); %pulse filter
  tx = filter(num, den, sum(x));               %sum pulses & filter
  tx = tx / norm(tx, 2);                       %make unit energy
end %make OTFS
```

Suppose we use the same path coefficients as before, but this time the second path is shifted by merely 0.1 milli-seconds delay and 2 Hz Doppler with respect to the first path:

```
%-----------------------------------------------------------------------
% STEP 1:  define channel paths for delay-Doppler multi-path channel
%-----------------------------------------------------------------------
tau = [0    0.1E-3];                        %delay in (s)
fd =  [0    2];                             %Doppler in (Hz)
a =   [1E-6 0.5E-6];                        %path coefficients (complex)
```

As an experiment, create the enhanced OTFS waveform, pass it through the channel, and report the signal energy as follows:

```
%-----------------------------------------------------------------------------
%  STEP 2:  Create OTFS Signal, Pass Through Channel, & Measure Energy
%-----------------------------------------------------------------------------
Fs = 1E6;                                 %sampling rate (samples/s)
Tsym = 2;                                 %transmission duration (s)
tx = makeOTFS(tau, fd, a, Fs, Tsym);      %build enhanced OTFS waveform
tx = fliplr(conj(tx));                    %time-reversed & freq. reversed
rx = channelModel(tau, fd, a, Fs, tx);    %pass thru channel
reportEnergy(tx, rx, 'OTFS');             %report results
```

The channel model applies delays, Doppler, magnitude adjustment, and phase rotations:

```
function y = channelModel(tau, fd, a, Fs, x)
%-----------------------------------------------------------------------------
% y = channelModel(tau, fd, a, Fs, x)
%
% Propagates input signal through a model 2-path multi-path channel
%
% Variable  Size     I/O  Description
% --------  -------- ---  ----------------------------------------
% tau       1x2      In   Delay for each of two paths in (s)
% fd        1x2      In   Doppler for each of two paths in (Hz)
% a         1x2      In   Path coefficient for each of two paths
% Fs        1        In   Sampling rate in (samples/s)
% x         Varies   In   Row vector of channel input time-series
% y         Varies   Out  Row vector of channel output time-series
%
%                                         -Pete Wyckoff, Version 06172025.
%-----------------------------------------------------------------------------
  hLen = round(max(tau)*Fs) + 2;                   %length for impulse resp.
  for m=1:2                                        %loop through 2 paths
    h = zeros(1, hLen);                            %initialize as all zeros
    h(round(tau(m)*Fs) + 1) = a(m);                %path delay & coefficient
    p = conv(x, h);                                %apply delay & coef.
    lo = exp(1i*2*pi*fd(m)*(0:length(p)-1)/Fs);    %path Doppler
    y(m,:) = p .* lo;                              %apply path Doppler
  end
  y = sum(y);                                      %sum the 2 paths
end %channelModel
```

The function “reportEnergy” computes the energy a prints a message to the terminal window:

```
function E = reportEnergy(tx, rx, idStr)
  txEnergy_dB = 10*log10(sum(abs(tx).^2));
  rxEnergy_dB = 10*log10(sum(abs(rx).^2));
  if(nargout == 0)
    fprintf('\n%s Transmitted Energy = %.1f (dB)', idStr, txEnergy_dB);
    fprintf('\n%s Received Energy = %.1f (dB) \n', idStr, rxEnergy_dB);
  end
  E = [txEnergy_dB, rxEnergy_dB];
end %reportEnergy
```

As a control group, suppose we test the same channel using a PN sequence waveform using similar time and bandwidth:

```
%-------------------------------------------------------------------------
%  STEP 3:  Create PN Signal, Pass Through Channel, & Measure Energy
%-------------------------------------------------------------------------
Fchip = 10E3;                                  %chip rate (chips/s)
tx = makePN(Fchip, Fs, Tsym);                  %create a PN signal
tx = fliplr(conj(tx));                         %time-reversed & freq. reversed
rx = channelModel(tau, fd, a, Fs, tx);         %pass thru channel
reportEnergy(tx, rx, 'PN');                    %report results
```

The function “makePN” produces this signal as follows:

```
function tx = makePN(Fchip, Fs, Tsym)
%-------------------------------------------------------------------------
% tx = makePN(Fchip, Fs, Tsym)
%
% Creates a root-raised cosine PN sequence waveform
%
% Variable  Size     I/O  Description
% --------  -------  ---  -----------------------------------------------
% Fchip     1        In   Chip rate in (chips/s)
% Fs        1        In   Samp. rate (samples/s) [Integer mult of Fchip!]
% Tsym      1        In   Symbol duration in (s)
% tx        Varies   Out  Row vector of transmit signal as time-series
%
%                                        -Pete Wyckoff, Version 06172025.
%-------------------------------------------------------------------------
  chips = sign(randn(1, round(Tsym*Fchip)));
  [num, den] = rcosine(Fchip, Fs, 'sqrt', 0.4);      %design filter
  tx = filter(num, den, upsample(chips, Fs/Fchip));  %apply filter
  tx = tx / norm(tx, 2);                             %make unit energy
end %makePN
```

The overall script reports the following to the terminal window:

```
OTFS Transmitted Energy = -0.0 (dB)
OTFS Received Energy = -117.6 (dB)

PN Transmitted Energy = -0.0 (dB)
PN Received Energy = -119.0 (dB)
```

This matches results from earlier, albeit using much smaller delay and Doppler along with a 2 second symbol duration. These make waveform design more complicated yet the new enhancement to OTFS still improves energy transfer efficiency +1.4 dB versus a PN waveform that is not designed for the channel.

(*Note: this enhanced OTFS waveform is certainly not optimized. For this particular channel, one could change the frequency of each pulse to boost the efficiency further. One could alternatively design an LFM that would boost the energy efficiency. However, such approaches do not extend well to three or more paths in general, so these are not presented here.*)

Imperfect Phase Knowledge in Two-Path Delay-Doppler Channel

The enhanced OTFS waveform takes advantage of the delay-Doppler channel through superposition of fields that were emitted at different times into one time-of-arrival at the receiver. This boosts received energy. It also makes the approach sensitive to phase errors in the transmitter's estimate of the channel. Figure 6 compares the new OTFS enhanced waveform to a genetic PN waveform as the unbiased random phase error on each path estimate increases. The results are for the two-path channel with the second path shifted 0.1 milliseconds in delay and 2 Hz in Doppler with respect to the first path. With no phase error, the new OTFS extension transfers energy from transmitter to receiver +1.4 dB more efficiently than generic PN. The boost gradually degrades as the phase error per path increases. At worse than 80-degrees standard deviation, the new OTFS extension delivers the same energy as a generic PN signal.

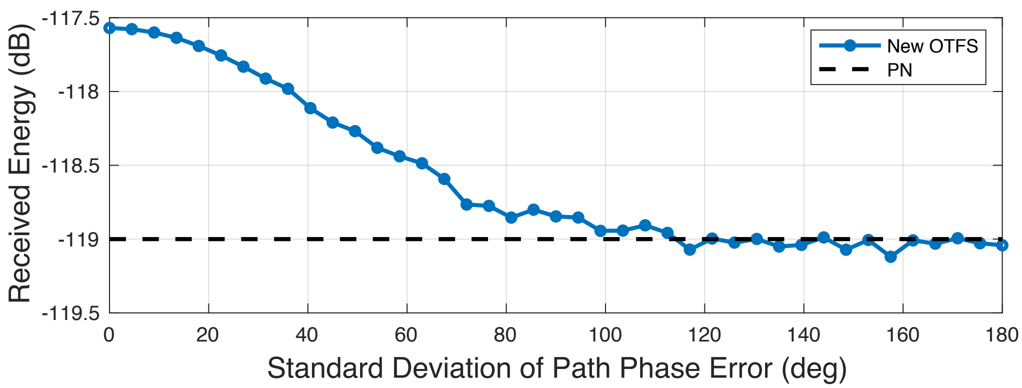

FIGURE 6: New OTFS benefits from accurate phase delivering energy up to +1.4 dB more efficiently vs. PN.

The following code produced Figure 6:

```
Fs = 1E6;                                %sampling rate (samples/s)
Tsym = 2;                                %transmission duration (s)
phaseStd = linspace(0, pi, 41);
TRIALS = 1000;
for test=1:length(phaseStd)
    test
  parfor trial=1:TRIALS
    tx = makeOTFS(tau, fd, a, Fs, Tsym); %build enhanced OTFS waveform
    tx = fliplr(conj(tx));               %time-reversed & freq. reversed
    aMod = a .* [exp(1i*phaseStd(test) *randn(1)), ... %phase error
                 exp(1i*phaseStd(test) *randn(1))];
    rx = channelModel(tau, fd, aMod, Fs, tx);          %pass thru channel
    E(trial,:) = reportEnergy(tx, rx, 'OTFS');         %report results
  end
  E = mean(10.^(E/10));                   %mean in (linear) units
  storeMeanTX_dB(test) = 10*log10(E(1));  %convert to (dB)
  storeMeanRX_dB(test) = 10*log10(E(2));  %convert to (dB)
end

figure;
subplot(211);
plot(rad2deg(phaseStd), storeMeanRX_dB);
```

Summary

For multi-path delay-Doppler channels, the efficiency of energy transferred from the transmitter to the receiver is a function of waveform design. A new extension of OTFS demonstrated one avenue to boost this energy efficiency as compared to a generic root-raised-cosine filtered PN waveform. For two paths with coefficients 1E-6 and 0.5E-6, the extension boosted energy transfer by +1.4 dB versus the generic signal. This performance advantage begins to erode when there is more than 20 degrees of error on the transmitter's knowledge of the path phases. For 80 degrees or more of such error, the designed waveform delivers about the same energy to the receiver as a generic waveform. Application gains from this approach depend upon channel estimation performance. As a result, the EME channel is particularly challenging due to 2.4 seconds of round-trip-delay and it requires further research into channel estimation approaches that are a separate problem from waveform design.

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About the Author

For more than 20 years, Peter S. Wyckoff has designed and tested a wide variety of digital communications systems, modems, and antenna arrays, particularly for the satellite communications industry. He graduated from Penn State University with an MSEE in 2000 and graduated from Pitt with a BSEE in 1997. Since graduation, he has been awarded seven U.S. patents, which are mostly about co-channel interference mitigation, antenna array signal processing, and digital communications. In May 2023, his textbook "Visualizing Signal Processing with Complex Values" earned Amazon's #1 best-selling signal processing new release.

References

[1] Efficiently Using Transmit Symbol Energy via Delay-Doppler Channels — Part I, Open Research Institute Inner Circle Newsletter, May 2025, p. 12-13.
[2] Efficiently Using Transmit Symbol Energy via Delay-Doppler Channels — Part I, Open Research Institute Inner Circle Newsletter, May 2025, p. 15-16.
[3] A. Monk, R. Hadani, Mi. Tsatsanis, S. Rakib, OTFS — Orthogonal Time Frequency Space, https://arxiv.org/pdf/1608.02993
[4]. Efficiently Using Transmit Symbol Energy via Delay-Doppler Channels — Part I, Open Research Institute Inner Circle Newsletter, May 2025, p. 16.