Efficiently Using Transmitted Symbol Energy via Delay-Doppler Channels — Part I

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Introduction

There are few DSP avenues to improve link margin for extremely weak signal amateur radio communications. Prior research into modulation and coding already efficiently uses energy that actually reaches the demodulator. This is not the only problem though. A separate and important problem via linear time-variant (LTV) multi-path channels is making efficient use of the constrained energy-per-symbol transmitted by an emitter.

This paper introduces an efficiency measure as the ratio of energy-per-symbol reaching the receiver (E_s) vs. energy-per-symbol leaving the transmitter antenna (E_{tx}). This is used to quantify advantages of DSP waveform design. For a single path line-of-sight channel, this metric in dB equals the receiver antenna gain minus the path-loss. For a single path channel, the specific transmit waveform shape does not matter. For a channel with more than one path, the efficiency metric becomes a function of the specific transmit waveform. As a result, some DSP waveform designs more efficiently transfer energy from the transmitter to the receiver — *exploiting rather than mitigating multi-path*. The goal is developing theory to inform future over-the-air amateur radio experiments possibly via the LTV Earth-Moon-Earth (EME) channel.

General Problem Statement & Performance Metric Definition

Figure 1 shows the model for this problem. The transmitter antenna emits $x(t)$ as a function of time. This signal undergoes path loss and passes through the channel with M-paths as represented by impulse response $h_M(t)$. After the receiver antenna gain, the signal is $y(t)$.

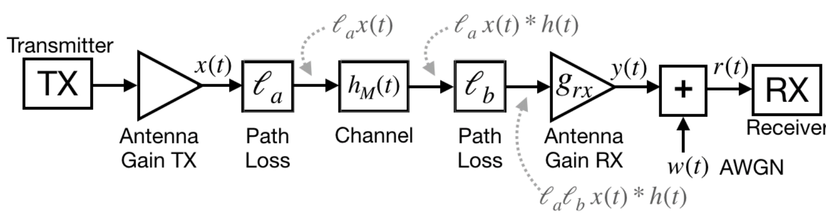

FIGURE 1: This system model for the problem is formulated generally as a moon-bounce (EME) channel yet it can be applied to many problems including direct line-of-sight.

The transmit symbol occurs during $0 \leq t < T_s$ where T_s is the symbol duration. Transmit symbol energy equals E_{tx} as the power integrated over time:

$$E_{tx} = \int_0^{T_s} |x(t)|^2 \, dt. \qquad \text{(Equation 1)}$$

Suppose the channel impulse response $h_M(t)$ paths deliver minimum path-delay τ_{min} and maximum τ_{max} during $0 \leq t < T_s$. Received symbol energy E_s equals:

$$E_s = \int_{\tau_{min}}^{T_s+\tau_{max}} |y(t)|^2 \, dt. \qquad \text{(Equation 2)}$$

Let the efficiency metric for the M-path LTV channel equal ξ_M:

$$\xi_M = 10\, log_{10}\left(\frac{E_s}{E_{tx}}\right). \quad \text{(Equation 3)}$$

This metric ξ_M is useful to analyze waveform efficiency for single-path and multi-path LTV channels.

Single-Path Efficiency Metric Analysis

The main point of this section shows waveform design does not impact the efficiency metric for a single-path channel. The single path channel has M=1 and therefore the channel impulse response exhibits path-delay τ_1 and path-coefficient α_1:

$$h_1(t) = \alpha_1 \delta(t - \tau_1). \quad \text{(Equation 4)}$$

From Figure 1, the receive antenna output equals:

$$y(t) = \ell_a \ell_b g_{rx} x(t) * h_1(t) = \ell_a \ell_b g_{rx} \alpha_1 x(t - \tau_1). \quad \text{(Equation 5)}$$

Suppose we summarize the path loss terms using $\ell = \ell_a \ell_b$. The received energy-per-symbol equals:

$$E_s = |\ell \alpha_1 g_{rx}|^2 \int_{\tau_1}^{T_s+\tau_1} |x(t-\tau_1)|^2\, dt = |\ell \alpha_1 g_{rx}|^2 \int_0^{T_s} |x(t)|^2\, dt. \quad \text{(Equation 6)}$$

The efficiency metric for a single path equals:

$$\xi_1 = 10\, log_{10}\left(\frac{E_s}{E_{tx}}\right) = 10\, log_{10} |\ell \alpha_1 g_{rx}|^2. \quad \text{(Equation 7)}$$

> **Example:** Suppose that for a single-path channel $\ell_a = \ell_b = 1 \times 10^{-3}$. The path loss terms alone equal $-10\, log_{10} |\ell|^2 = 120$ dB. Suppose we further set $g_{rx} = 1$, and $\alpha_1 = 1$. Therefore $|\ell \alpha_1 g_{rx}|^2 = 1 \times 10^{-12}$ and the efficiency factor equals -1 times the path loss as $\xi_1 = 10\, log_{10}\left(1 \times 10^{-12}\right) = -120$ dB.

In summary, efficiency shows that less path loss improves efficiency. As well, greater receiver antenna gain improves efficiency. If the path is reflected — *moon-bounce for instance* — then a stronger reflection $|\alpha_1|$ also improves efficiency. The key-point of Equation 7 though is the efficiency for a single path is not a function of the transmitted waveform shape $x(t)$. Provided two different alternatives for the shape of $x(t)$ have the same E_{tx}, then either alternative delivers the same E_s for the single-path channel formulation of Figure 1.

Multi-path Efficiency Metric Analysis

The main point of this section shows that with more than one path (M>1), the efficiency metric is impacted by the shape of the transmitted waveform $x(t)$. For an LTV channel with M paths, the channel impulse response equals $h_M(t)$:

$$h_M(t) = \sum_{m=1}^{M} \alpha_m \delta(t - \tau_m - \dot{\tau}_m t) \quad \text{(Equation 8)}$$

The m^th path has magnitude and phase α_m, path delay τ_m at time $t = 0$, and rate-of-change in path delay $\dot{\tau}_m$. If the channel were linear time-invariant (LTI), then it would have $\dot{\tau}_m = 0$ for all paths. Since the channel scattering — *amateur radio moon bounce for instance* — has relative motion, the linear time-variant (LTV) channel is a more general formulation. It allows for motion. The path delay $\tau_m = r_m / c$ for a path with range r_m meters and wave propagation speed c meters/second. The rate-of-change $\dot{\tau}_m = v_m / c$ for a path with range-rate v_m meters/second.

Using this channel, the received energy-per-symbol equals:

$$E_s = \int_{\tau_{min}}^{T_s+\tau_{max}} |y(t)|^2 dt = |\ell g_{rx}|^2 \int_{\tau_{min}}^{T_s+\tau_{max}} |x(t) * h_M(t)|^2 dt. \quad \text{(Equation 9)}$$

The efficiency metric equals for M>1 paths equals ξ_M:

$$\xi_M = 10\, log_{10} \left(|\ell g_{rx}|^2 \frac{\int_{\tau_{min}}^{T_s+\tau_{max}} |x(t) * h_M(t)|^2 dt}{E_{tx}} \right). \quad \text{(Equation 10)}$$

For the multi-path LTV channel, the shape of the waveform $x(t)$ impacts the efficiency metric. This characteristic makes the multi-path channel different than the single-path channel.

Using Time-Lens Effect of a Known LTI Multi-Path Channel to Boost Efficiency Metric

As a baseline, Figure 2 passes a pulsed RF signal through a multi-path LTI channel without range-rate — *in other words* $\dot{\tau}_m = 0$. For this example, E_{tx} = 0.0 dB and E_s = -119.0 dB as shown in the figure. The efficiency metric is therefore -119.0 dB for this first waveform.

For comparison, Figure 3 passes a different signal using two RF pulses through the same channel. Whereas E_{tx} = 0.0 dB still, now E_s = -117.6 dB as shown in the figure. For this second waveform, the efficiency metric is boosted to -117.6 dB. This waveform, using two pulses, is 1.4 dB more efficient than the single pulse waveform for this LTI channel.

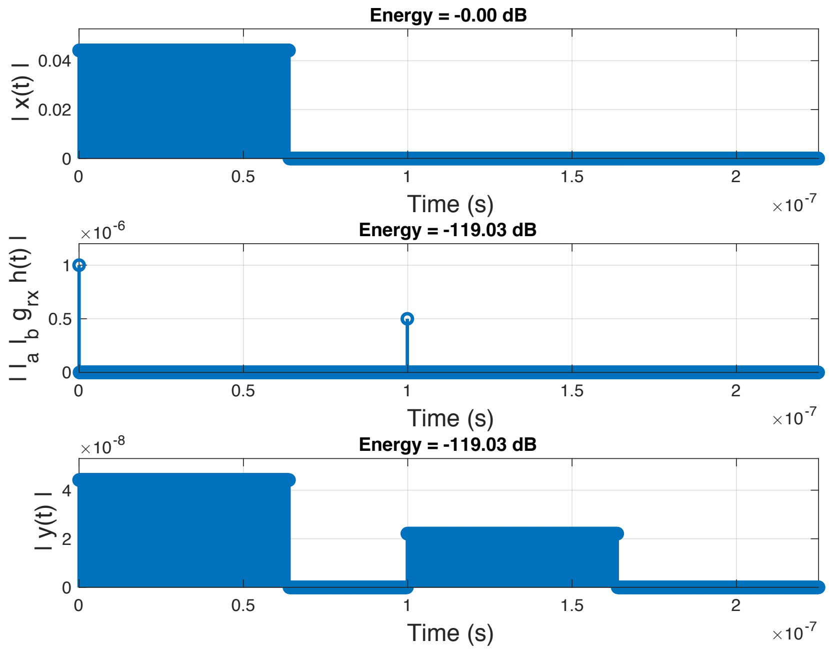

FIGURE 2: A single pulse waveform yields efficiency metric -119.03 dB.

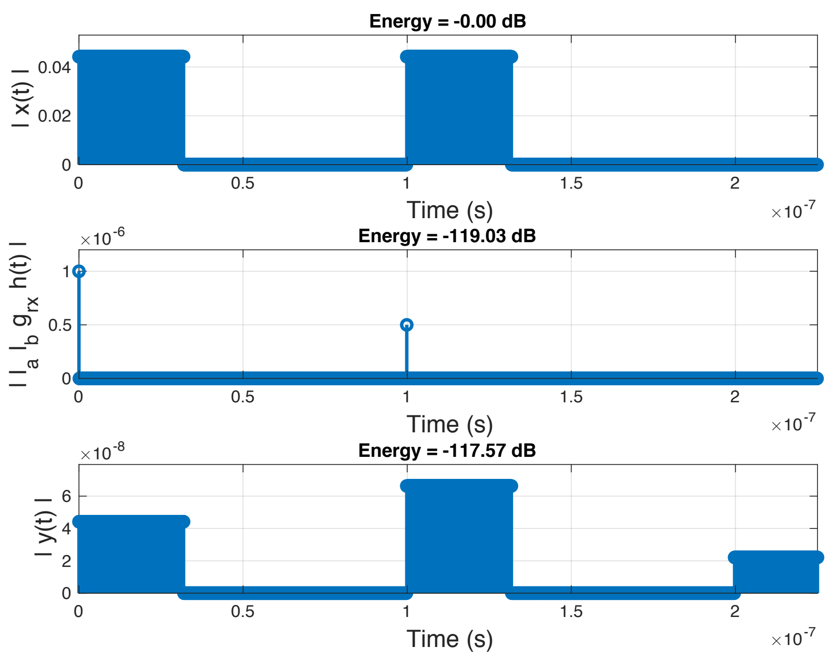

FIGURE 3: A two pulse waveform yields efficiency metric -117.57 dB.

For this channel, each input pulse produces two output pulses. In Figure 3, two input pulses delivered only three output pulses. During the middle pulse, two pulses that were transmitted at different times constructively combined to boost the field-strength. Since power is proportional to the magnitude of field-strength squared, the received waveform delivers more energy to the receiver than some other transmit waveform that does not combine pulses in this manner.

Figure 4 boosts the efficiency further by using 8 pulses. E_{tx} = 0.0 dB still, and E_s = -117.22 as shown in the figure. The efficiency metric is now -117.22 dB. For this channel, using these 8 pulses is 1.78 dB more efficient than the single pulse from Figure 2.

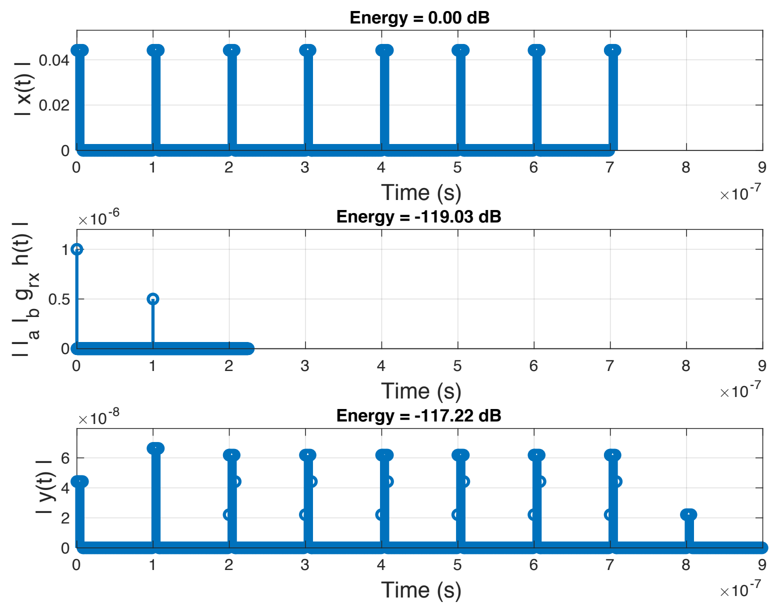

FIGURE 4: An eight pulse waveform yields efficiency metric -117.22 dB.

Since this example uses $\alpha_1 \ell g_{rx} = 1 \times 10^{-6}$ and $\alpha_2 \ell g_{rx} = 0.5 \times 10^{-6}$, the efficiency of the single short pulse from Figure 2 will be the sum of the energy delivered by each path:

$$10\, log_{10}((\,|\,(1 \times 10^{-6}\,|^2 + |\,0.5 \times 10^{-6}\,|^2\,)/1) = -\,119 \text{ dB.} \qquad \text{(Equation 11)}$$

This represents the gain from multi-path with a signal that adds in power only. The efficiency from a train of many designed pulses will be the square of the magnitude of the sum of the path coefficients, which is larger:

$$10\, log_{10}(\,|\,(1 \times 10^{-6} + 0.5 \times 10^{-6}\,|^2\,/1) = -\,116.4 \text{ dB.} \qquad \text{(Equation 12)}$$

This represents the gain from multi-path with a signal that adds in field-strength. Thus, a very long train of many designed pulses will be up to 2.6 dB more efficient than the single pulse for this channel. Of course if there are errors in estimating the channel response, the efficiency gain will be less — *more to follow in later Part.*

Summary of Part I

Part 1 presented a waveform efficiency metric. Given a known LTI multi-path channel, waveform design examples were shown that improved the waveform efficiency metric. The particular waveforms were designed as pulsed waveforms because these serve as a foundation to build waveforms for the LTV channel in Part 2. Those waveforms use techniques from Orthogonal Time Frequency Space (OTFS) to take similar advantage of delay-Doppler channels. The goal for this series of papers is developing theory to inform future over-the-air amateur radio experiments possibly via the LTV Earth-Moon-Earth (EME) channel.